Recent progress in algebraic combinatorics
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1. The Laurent phenomenon. There are many examples of rational functions
with recursive definitions which turn out to be Laurent polynomials. The proto-
typical example is the (generic) Somos-4 sequence, defined by

\[ a(0) = a, a(1) = b, a(2) = c, a(3) = d \]
\[ a(n)a(n + 4) = a(n + 1)a(n + 3) + a(n + 2)^2, \quad n \geq 0, \]

where \( a, b, c, d \) are indeterminates. A breakthrough in understanding the Lau-
rent phenomenon algebraically was made by Fomin and Zelevinsky as a conse-
quence of their theory of cluster algebras.

2. Toric Schur functions and Gromov-Witten invariants. Three-point Gromov-
Witten invariants are a generalization of the classical intersection numbers
(given by Littlewood-Richardson coefficients) of Schubert calculus. They may
be described by means of a quantum deformation \( \mathbb{Q}H \) of the cohomology ring
\( H \) of the Grassmann variety. A basis for \( H \) consists of Schur functions indexed
by certain Young diagrams. Recently Alexander Postnikov extended this de-
scription to \( \mathbb{Q}H \) by defining toric Schur functions whose corresponding Young
diagram lies on a torus rather than in the plane. This yields a new method
for computing Gromov-Witten invariants which resolves several open problems
concerning them, as well as leading to some new properties.