

Recent progress in algebraic combinatorics

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1. The Laurent phenomenon. There are many examples of rational functions with recursive definitions which turn out to be Laurent polynomials. The prototypical example is the (generic) Somos-4 sequence, defined by

$$a(0) = a, a(1) = b, a(2) = c, a(3) = d$$

$$a(n)a(n+4) = a(n+1)a(n+3) + a(n+2)^2, n \geq 0,$$

where a, b, c, d are indeterminates. A breakthrough in understanding the Laurent phenomenon algebraically was made by Fomin and Zelevinsky as a consequence of their theory of cluster algebras.

2. Toric Schur functions and Gromov-Witten invariants. Three-point Gromov-Witten invariants are a generalization of the classical intersection numbers (given by Littlewood-Richardson coefficients) of Schubert calculus. They may be described by means of a quantum deformation QH of the cohomology ring H of the Grassmann variety. A basis for H consists of Schur functions indexed by certain Young diagrams. Recently Alexander Postnikov extended this description to QH by defining toric Schur functions whose corresponding Young diagram lies on a torus rather than in the plane. This yields a new method for computing Gromov-Witten invariants which resolves several open problems concerning them, as well as leading to some new properties.